University of California, Berkeley
Physics H7C Fall 2002 (*Strovink*)

## PROBLEM SET 4

**1.** *Shiny metals.*
For a metal, the right-hand side of the wave equation (proportional to the conductivity $\sigma$) does not vanish. The equation can still be solved by a sinusoid with angular frequency $\omega$ and phase velocity $v_{\text{phase}} = c/n$ as usual. Here, however, the refractive index $n$ is *complex*. In particular, in good conductors with $\mu = \mu_0$ and

$$\frac{\sigma}{\epsilon_0 \omega} \gg 1 ,$$

the complex index becomes

$$n \approx \sqrt{\frac{\sigma}{\epsilon_0 \omega}}\, e^{i\pi/4} .$$

Nothing that we did while deriving Fresnel's equations assumed that $n$ was real, so the derivation still is valid.
(a.)
Show that good-conducting metals are *shiny*, *i.e.* demonstrate that at normal incidence essentially all of the the incident beam intensity is reflected.
(b.)
Provide a rough estimate of how far the transmitted wave penetrates into the metal. (This is called the *skin depth*.)

**2.** *Flat cable.*
A TEM waveguide has two conductors, and fields that are transverse to its long axis (a coaxial cable is a good example). Consider a pulse travelling down a TEM waveguide. At any point in the pulse, the cable's *characteristic impedance* $Z$ is defined to be the ratio of the |potential difference| between the two conductors, divided by the |current| flowing in either conductor (the current flows out on one conductor and back on the other). $Z$ is equal to $\sqrt{L/C}$, where $L$ is the cable's inductance per unit length and $C$ is the capacitance per unit length.

Sometimes it's desirable for a TEM waveguide to have a very *low* characteristic impedance (for example, if a low-impedance load like a spark chamber must be driven). This can be accomplished by putting many coaxial cables in parallel. More elegantly, one could build a flat cable consisting of a thin sheet of dielectric sandwiched between two sheets of conductor. If the dielectric thickness is small compared to the cable's width, fringe fields can be neglected, and it is straightforward to calculate the flat cable's characteristic impedance.
(a.)
Given a ribbon of polyethylene with $\mu = \mu_0$, $n = 1.5$, and a thickness of 0.2 mm, design a flat cable with a characteristic impedance of 3.77 $\Omega$ (1% of the characteristic impedance of free space). What should the width of this cable be?
(b.)
You assign a subordinate to fabricate this flat cable. He builds part of it with the specified width, but, to avoid running out of material, he builds the remainder with only half the width that you specified. If you send a voltage pulse down the fat end of this mongrel cable, what fraction of it will reflect back from the junction between its mutually dissimilar parts?

**3.**
Right-hand circularly polarized light is incident from vacuum upon a semi-infinite slab of clear insulator with $\mu = \mu_0$ (so that Fowles' Eqs. (2.54-2.59) do apply) and with refractive index $n = \sqrt{3}$. The incident beam makes a 60° angle with the normal to the slab's surface.
(a.)
Describe the polarization of the reflected light.
(b.)
What fraction of the incident beam intensity is reflected?

**4.** *Mirage.*
On page 45, Fowles states *"For grazing incidence ($\theta \sim 90$ degrees), the reflectance is... the same for both types of polarization, namely, unity, and it is independent of $n$."* Let's analyze this

claim in a bit more detail. For the two media, we'll assume that $\mu_1 = \mu_2$, so that Fowles' Eqs. (2.58-2.59) do apply. In those equations, take $\theta = \frac{\pi}{2} - \psi$ and consider the limit as $\psi \to 0$.

(a.)

In that limit, show that the reflectance (= fraction of light energy that is reflected) indeed is unity and independent of $n = n_2/n_1$ for both types of polarization, as Fowles claims.

(b.)

In that same limit, show that the ratio of the reflected to the incident *amplitude* of $\vec{E}$ is *not* the same for both types of polarization, using the sign conventions implicit in Fowles' Fig. 2.11. Looking at that figure, is it reasonable for $\vec{E}\,'$ to "flip" relative to $\vec{E}$ for one type of polarization, but not for the other?

(c.)

Now consider the mirage to lowest *nonvanishing* order in $\psi \ll 1$. If you want to *minimize* your confusion from mirages, and you are wearing Polaroid sunglasses, how should their absorption axis be oriented relative to the horizon?

**5.**

Among the many celebrated inventions of Luis Alvarez, a member of the Berkeley Physics Department from the 1940's up to his death in the late 1980's, was the radar glide-path system for guiding an airplane's landing approach during bad weather. (This system was used first in the Battle of Britain, where it saved the lives of many Allied aviators.) The basic principle was similar to that of Lloyd's mirror (Fowles Fig. 3.3(a)): a radar transmitter was located at point $S$ and the ground itself served as the mirror (radar frequencies are so low that the ground is "shiny"; see Problem 1). The airplane at point $P$ carried a crude radar detector; the pilot adjusted his trajectory to maximize the radar signal.

(a.)

Argue that the lowest-elevation maximum in the interference pattern was *not* produced at ground level (which would have been a disaster).

(b.)

If the WWII radar wavelength was 1 m and the B-17 was to approach the ground at an angle of 0.1 rad (these numbers are speculative), at what height should the radar transmitter have been set?

**6.**

Calculate the interference pattern that would be obtained if three thin slits instead of two were used in Young's experiment (assume equal spacing of the slits). Assume further that the two outer slits are identical, but that the center slit (though still thin) passes twice as large an electric field amplitude as either of the outer slits.

**7.**

We wish to use the light of the sun (angular width 0.5°), passed through a 600 nm filter, as the source for a double-thin-slit Young's interference experiment.

(a.)

Assuming a very narrow filter bandpass, estimate the maximum slit separation (in mm) that would yield an interference pattern which isn't too badly washed out, *i.e.* with a fringe visibility $V \approx \frac{1}{2}$.

(b.)

Assuming an adequately small slit separation, roughly estimate the maximum filter bandpass (in nm) that would allow us to observe at least 20 fringes. With this choice of bandpass, what is the coherence length of the transmitted light?

**8.**

Fowles 3.12. Note that the "power spectrum" is $|g(\omega)|^2$, where $g(\omega)$ is the Fourier transform of $f(t)$.